Tina Memo No. 1999-003
Internal Report.

# Error Modelling of Stereo Vision Data.

A.Harris, N.A.Thacker.

Last updated
1 / 3 / 1998

WWW.TINA-VISION.NET

Imaging Science and Biomedical Engineering Division,
Medical School, University of Manchester,
Stopford Building, Oxford Road,
Manchester, M13 9PT.

# Error Modelling of Stereo Vision Data.

A.Harris, N.A.Thacker

## 1 Modelling Errors

### 1.1 Feature location errors

Feature based stereo vision algorithms can use either corner or edge data to extract depth information. The errors on the locations of these are due to the accuracy of the feature detector used, but tend to be of similar magnitude for each extraction algorithm.

For corner data, the error $\Delta x$ is typically 0.1 pixels.

For edge based data, the error in an individual edgel location is due to the accuracy of both the edge detector and the rectification process which determines the location of the epipolar.

$$\Delta x = \sqrt{\Delta x_f{}^2 + \Delta x_e{}^2}$$

where $\Delta x_f$ is the error in edge location (typically 0.1 pixels) and $\Delta x_e$ is the error due to the epipolar placement and is found using:

$$\Delta x_e = \frac{\Delta e}{\tan\theta}$$

where:

$\Delta e$ is the error in epipolar placement ($\approx 0.1$ pixels)
$\theta$ is the acute angle between the edge and the epipolar

as $\theta \rightarrow 0$ then $\Delta x_e \rightarrow \infty$ so we set a minimum value of $\theta$ for an edge to be used.

### 1.2 Propagating the feature error into the world

Using rectified images, the distance, $Z$ between the feature and the camera plane can be found with the equation:

$$Z = \frac{fI}{x_1 - x_2}$$

where:

$f$ is the focal length of the lenses
$I$ is the inter-occular separation
$x_1$ and $x_2$ are the positions of the features on the epipolars in each image

We can determine the sensitivity of $Z$ with changes in $x_1$ and $x_2$ thus,

$$\Delta Z^2 = \left(\frac{\delta Z}{\delta x_1}\Delta x_1\right)^2 + \left(\frac{\delta Z}{\delta x_1}\Delta x_2\right)^2$$

where,

$$\frac{\delta Z}{\delta x_1} = -\frac{fI}{(x_1 - x_2)^2} \quad \text{and} \quad \frac{\delta Z}{\delta x_2} = \frac{fI}{(x_1 - x_2)^2}$$

$\Delta x$ is the feature position error in the image and can be assumed to be equal in each image, so

$$\Delta x_1 = \Delta x_2 = \Delta x$$

Solving for $\Delta Z$ yields the result,

$$\boxed{\Delta Z = \frac{\sqrt{2} f I \Delta x}{(x_1 - x_2)^2} \quad \text{or in terms of } Z, \quad \Delta Z = \frac{\sqrt{2} Z^2 \Delta x}{f I}}$$

The world co-ordinates $X$ and $Y$ can be found using the equations:

$$X = \frac{Ix}{x_1 - x_2} \quad \text{and} \quad Y = \frac{Iy}{y_1 - y_2}$$

where $x$ and $y$ are image co-ordinates.

$x$ is correlated to $x_1$ and $x_2$, so we can perform error sensitivity using only $x_1$ and $x_2$

$$\Delta X^2 = \left(\frac{\delta X}{\delta x_1}\Delta x_1\right)^2 + \left(\frac{\delta X}{\delta x_1}\Delta x_2\right)^2$$

where,

$$\frac{\delta X}{\delta x_1} = -\frac{Ix}{(x_1 - x_2)^2} \quad \text{and} \quad \frac{\delta Z}{\delta x_2} = \frac{Ix}{(x_1 - x_2)^2}$$

$$\boxed{\Delta X = \frac{\sqrt{2} I x \Delta x}{(x_1 - x_2)^2} \quad \text{or in terms of } Z, \quad \Delta X = \frac{\sqrt{2} x Z^2 \Delta x}{f^2 I}}$$

similarly,

$$\boxed{\Delta Y = \frac{\sqrt{2} I y \Delta y}{(y_1 - y_2)^2} \quad \text{or in terms of } Z, \quad \Delta Y = \frac{\sqrt{2} y Z^2 \Delta y}{f^2 I}}$$

### 1.3 Transformation into disparity space

We can see from the equations above, that the error on the world co-ordinate estimate is not uniform in the directions $X, Y$ and $Z$. To make statistical measurements in the world space, we need the error to be uniform in all directions. This can be accomplished by transforming the world space $P(X,Y,Z)$ into disparity space $P'(X',Y',Z')$.

$$X' = \frac{x}{\Delta x} \quad ; \quad Y' = \frac{y}{\Delta x} \quad ; \quad Z' = \frac{x_1 - x_2}{\sqrt{2}\Delta x}$$

By performing error sensitivity in this new space, we see that the error is unity in each direction.

$$\Delta X' = \left(\frac{1}{\Delta x}\right).\Delta x = 1$$
$$\Delta Y' = \left(\frac{1}{\Delta y}\right).\Delta y = 1$$
$$\Delta Z' = \left[\left(\frac{\Delta x}{\sqrt{2}\Delta x}\right)^2 + \left(\frac{-\Delta x}{\sqrt{2}\Delta x}\right)^2\right]^{\frac{1}{2}} = 1$$

We can also describe disparity space in terms of the world co-ordinate system by substitution of the image co-ordinates.

$$X' = \frac{fX}{\Delta x Z} \quad ; \quad Y' = \frac{fY}{\Delta x Z} \quad ; \quad Z' = \frac{fI}{\sqrt{2}\Delta x Z}$$

To use the stereo algorithm in an obstacle avoidance system, we need to estimate the world distance between a feature and the ground plane. To achieve this we need to determine the equation of the ground in world co-ordinates, then transform the plane into disparity space to find the distance in standard deviations of the uniform error.

If the equation of the plane is:

$$aX + bY + cZ = D$$

then this transforms to:

$$a\left(\frac{IX'}{\sqrt{2}Z'}\right) + b\left(\frac{IY'}{\sqrt{2}Z'}\right) + c\left(\frac{fI}{\sqrt{2}\Delta x Z'}\right) = D$$

this rearranges to:

$$aI\Delta x X' + bI\Delta x Y' - \sqrt{2}\Delta x D Z' = -cfI$$

which is a plane of the form:

$$a'X' + b'Y' + c'Z' = D'$$

where $a' = aI\Delta x$ ; $b' = bI\Delta x$ ; $c' = -\sqrt{2}\Delta x D$ ; $D' = -cfI$