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Introduction

The process of non-maximal suppression is investigated here with the intention of obtaining a quantitative expression for the effects of independant random noise. This theoretical work is generated as part of the 3D recogntion project and requires experimental validation.

Analysis

Edges are located on a significance image map which has high values over the required features. We define the process of edge detection as being the identification of pixels which have a grey level g above a threshold value t and a value greater than at least 6 of its eight neighbours on such maps. This process defines connected ridges within the significance map. We need to understand how the noise on the input image data affects the identification of edge pixels. We can construct such significance maps in several ways, but can reasonably assume that the errors on the significance estimates is identical independent random (IID) for values greater than the required detection threshold.

We will compute the probability that an edge will be detected within a pixel P from an analysis of the probability of detecting a local maxima followed by the application of the threshold. The probability that a given grey level g_i is above a specific value x is given by;

$$P(g_i > x) \ = \ P_i \ = \ \int_x^\infty p(g_i, y) \ dy$$

where $p(g_i, y)$ is the probability density as a function of grey level y.

The probability a value g is larger than at least six of its eight neighbours $P(g > g_i \in R_6)$. the latter is given by;

$$P(g > g_i \in R_6) \ = \ P_R \ = \ \frac{1}{2}\sum_j^8 \sum_{k \neq j}^7 \Pi_{i \neq j,k}^6 \ P_i \tilde{P}_j \tilde{P}_k \ + \ \sum_j^8 \Pi_{i \neq j}^7 \ P_i \tilde{P}_j \ + \ \Pi_i^8 \ P_i$$

where P_i and $\tilde{P}_i$ are the probability of g being greater than g_i and it complement, $P_i \ = \ 1 - \tilde{P}_i$. The three terms correspond to the probability that the grey level is greater than 6 of the neighbours, 7 of the neighbours and all of the neighbours respectively. Though this is already the correct definition for a ridge detection process, the computational form is not particularly convenient for manipulation or numerical evaluation. Particularly, as the expressions involve nested loops due to the combinatorial from of the probability definitions. The following section attempts to rewrite this expression is a form which is more concise for manipulation and implementation.

We can rewrite this as;

$$= \ \frac{1}{2}\sum_j^8 \sum_k^7 \Pi_i^8 \ P_i \frac{\tilde{P}_j}{P_j}\frac{\tilde{P}_k}{P_k} \ + \ \sum_j^8 \Pi_i^8 \ P_i \frac{\tilde{P}_j}{P_j} \ + \ \Pi_i^8 \ P_i$$

Taking out the common factor $\Pi_i^8 \ P_i$ gives;

$$P_R \ = \ \Pi_i^8 \ P_i \ \left[\frac{1}{2}\sum_j^8 \sum_{k \neq j}^7 \frac{\tilde{P}_j}{P_j}\frac{\tilde{P}_k}{P_k} \ + \ \sum_j^8 \frac{\tilde{P}_j}{P_j} \ + \ 1 \right]$$

$$= \ \Pi_i^8 \ P_i \ \left[\sum_j^8 \frac{\tilde{P}_j}{P_j} (\frac{1}{2} \sum_{k \neq j}^7 \frac{\tilde{P}_k}{P_k} \ + \ 1) \ + \ 1 \right]$$

$$= \ \Pi_i^8 \ P_i \ \left[\sum_j^8 \frac{\tilde{P}_j}{P_j} (\frac{1}{2} \sum_k^8 \frac{\tilde{P}_k}{P_k} \ + \ 1 \ - \ \frac{1}{2}\frac{\tilde{P}_j}{P_j}) \ + \ 1 \right]$$

$$= \ \Pi_i^8 \ P_i \ \left[\sum_j^8 \frac{\tilde{P}_j}{P_j} (\frac{1}{2} \sum_k^8 \frac{\tilde{P}_k}{P_k} \ + \ 1) \ - \ \frac{1}{2}\sum_k^8 \frac{\tilde{P}_j^2}{P_j^2} \ + \ 1 \right]$$

Now introducing the combined variables;

$$J(\mathbf{g}) \;=\; \sum_j^9 \frac{\tilde{P}_j}{P_j} \;\;,\;\; K(\mathbf{g}) \;=\; \sum_k^9 \frac{\tilde{P}_k^2}{P_k^2} \;\; and \;\; L(\mathbf{g}) \;=\; \Pi_i^9 \, P_i$$

which now also include the central pixel value as the 9th term g_9 in the vector of measurements $\mathbf{g}$ such that;

$$\sum_j^8 \frac{\tilde{P}_j}{P_j} \;=\; J \;-\; 1 \;\;,\;\; \sum_k^8 \frac{\tilde{P}_k^2}{P_k^2} \;=\; K \;-\; 1 \;\;\; and \;\;\; \Pi_i^8 \, P_i \;=\; 2 \, L$$

We can now write;

$$\begin{aligned} P_R(\mathbf{g}) \;&=\; 2 \, L \left[(J \;-\; 1)(\frac{1}{2}(J \;-\; 1) \;+\; 1) \;-\; \frac{1}{2}(K \;-\; 1) \;+1 \right] \\ &=\; 2 \, L \left[\frac{1}{2}J^2 \;-\; J \;+\; \frac{1}{2} \;+\; J \;-\; 1 \;-\; \frac{1}{2}K \;+\; \frac{1}{2} \;+\; 1 \right] \\ &=\; L \, \left[J^2 \;-\; K \;+\; 2 \right] \end{aligned}$$

This form of the expression is now much easier to implement compute than the original, as it contains terms computed from simple loops over the data. Although it should be noted that both the L term and the K term are needed to cancel potential infinities in J, so some care must be taken in any numerical evaluation.

We must now assess the effect of thresholding the central pixel value. The probability that a candidate local maximum will be above the required threshold t at the central value g is given by the expectation value over the possible values for the central pixel g_9 in the allowed range;

$$P_e \;=\; \int_t^\infty p(g_9, g) P_R(\mathbf{g}) \; dg_9 \;=\; \int_t^\infty p(g_9, g) L \; \left[J(\mathbf{g})^2 \;-\; K(\mathbf{g}) \;+\; 2 \right] \; dg_9$$

So that the probability can be computed from sums or products of terms involving the 3x3 region surrounding the test pixel. We assert that this probability is now suitable for use in a likelihood analysis of IID significance map data.