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# A Methodology for Constructing View-Dependent Wireframe Models

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## Abstract

This paper outlines a strategy for representing and localising rigid three dimensional objects in images using edge data. A probabilistic metric combining edge strength and corresponding orientation information is developed to support this process. A process of lateral feature shifting in the image plane is presented to quantitatively account for a range of modelling/ illumination dependencies. We show that this mechanism is required to get good agreement between model and image data.

## 1 Introduction

The use of edge features for object detection and localisation tasks is prevalent throughout the history of computer vision [9, 10, 11]. The contours of objects, as defined by projected edges, concisely convey the bulk of image information pertaining to object shape, identity and position. Such features also offer a high degree of invariance to changes in background and environmental illumination. This latter property bypasses some significant problems, otherwise encountered when using appearance-based techniques, in having to account for every possible way that an object may be presented and illuminated.

For this work, a stereo computer vision system is used to provide initial data in the form of a 3D edge-based depth map. These edges are then converted to line and elliptical (typically circular) features. Representative wireframe object models are constructed from similar features representing each object's defining edges. Initial matching is performed by searching for sets of three dimensionally distributed features with the expected geometric configurations. Predicted edge features are projected for comparison with image evidence, computed in the form of a Likelihood score. This allows the model to be optimally aligned with any detectable features and any associated camera parameters to be determined. However, our previous work [1] highlighted problems with edge features appearing away from their predicted positions, thus impairing the ability to reliably localise or verify the presence of any such object models. These issues were attributed to variations in image content due to interactions between arbitrary scene illumination and object complexity.

Despite the problem reduction facilitated by utilising edge features, many of an objects' defining edge features may have some degree of positional variability with regard to variations in illumination. This current research identifies how to adapt wireframe models to account for this and other dependencies.

In this paper, a quantitative metric is formulated to support adaptive localisation of detectable edge features in the image plane. We show that by accounting for variations in detected edge location, this process supports more informed model match hypothesis verification and consequently more accurate 3D object localisation. The research was undertaken to enhance the associated capabilities of the TINA model matching stereo computer vision system [1]. Although, in this work, a full 3D wireframe is used directly for model matching, such a representation can further be utilised for view sampling. The intention here is to go on to use these models to support a view-based 3D object recognition system based upon geometric hashing for object indexing [2].

## 2 Methods

### 2.1 Wireframe Object Modelling

In order to detect a 3D object in a scene, we require a model detailing how that object may appear from any direction across the view sphere. Ideally, such representative models would be learnt automatically by a computer vision system [12]. Commonly, representative models are manually composed by the vision programmer. This process however conveniently avoids complications associated with the automated handling and inspection of 3D objects. Furthermore, many edge features may not be visible under certain illumination conditions, so that an object may need to be otherwise examined under a range of illuminations for each sampled viewpoint.

Given the current trend towards convergence of the topics of graphics and computer vision, perhaps the most obvious approach to modelling an object might be to employ a CAD-type model, detailing all aspects of an object's surfaces [8, 14]. However, as the primary goal is to ultimately construct and update the model using visual information, this raises a fundamental problem that is at the very core of the difference between these two topics.

CAD models support realistic rendering of scenes only because we can demand knowledge of all the required parameters with unlimited precision, including the effects of illumination. In computer vision, we cannot reliably represent the data from an image using CAD-like surfaces as there is never enough information to uniquely extract them. In general, there is little, if any, information visually conveyed by a smooth image region with which to parameterise any corresponding surface. Previous authors have sought to avoid this problem by constructing models from multiple views of an object [12], requiring both prior selection of suitable images and control over illumination. Such a process represents a highly restrictive data generation mechanism for robotic applications. Surface-based reconstruction further requires detailed illumination models, while edge-based reconstructions will be affected if (as we illustrate below) the apparent relative positions of detected edges actually move under object rotation or changes in illumination. We therefore follow an appearance modelling philosophy, and extract detectable image features in the places they can be detected, rather than attempting to extract surfaces.

One problem with not directly modelling objects’ surfaces is that edge feature visibility is directly related to surface occlusion. The visibility of a model’s edge feature points can be determined by verifying that a ray from each point to the camera does not intersect a surface. In this work, viewpoint dependency information is instead learned from image data. This data is stored in files, along with each wireframe, to indicate which features should be visible for specific viewpoints. The nearest stored viewpoint to one specified can then be taken to indicate which features should be visible. Although more intricate 3D objects may require more viewpoints for faithful 3D mapping, eight equidistant viewpoints has been sufficient in this work to relatively faithfully account for an object’s appearance. Partial feature visibility, due to occlusion, can be approximately represented by, for instance, splitting features to a desired degree of precision. While this may result in slight inaccuracies in comparison to CAD-based visibility processes, it is suggested that the associated convenience outweighs any drawbacks. We set a goal of modelling 90% of an object’s defining features, which we believe should be sufficient for localisation and verification, when used appropriately.

There are two main classes of features defining a 3D object’s shape; **fixed** edge features (such as sharp planar surface discontinuities) and **viewpoint dependent** ones (representing the extremal projected boundaries of any continuous surfaces). While the first class of edge features can simply be projected into the image from 3D, the second class presents more of a problem. To represent such features for arbitrary objects, a volumetric model is required from which to determine any such boundaries [13]. In this work, mostly for convenience, we are concerned with the modelling and detection of man-made objects with well-defined simple geometrical structure. We assume that we can therefore account for a broad range of objects with conical and cylindrical sections. The viewpoint dependent outer profiles of any such shapes can be accounted for by connecting the points on the end ellipses that are most distant from the axis connecting the two ellipses’ midpoints. More complicated structures can be approximated, to any required degree of accuracy, using sets of object boundaries and resulting curves.

An object model must also account for image scale. The features describing an object’s shape will be dependent on image scale, with finer features not being visible at low scales. Edge detection is also notoriously inaccurate at low resolution, where the predicted position of a step edge may be significantly distorted. Another scale related consideration is that of thin plate representation. Unless infinitesimally thin, a planar region will have back and front boundaries of surface discontinuity. Representing both, especially relative to low scale imagery, would clearly be unnecessary. Instead, a single feature could be modelled, which could be adaptively fitted, as will now be described, against the most prominent corresponding image edge data. This accords with the general philosophy of developing simple generalisable wireframe models.

To support optimised model localisation and verification, we have previously used an ‘edge potential map’ ( $V(x, y)$ ) to approximate the Likelihood of each image pixel corresponding to an edge. An optimisation function such as ‘simplex’ can be utilised to best align the projected 3D features, while simultaneously calibrating any associated camera parameters [1]. A number of studies have highlighted the benefits to be gained from using edge orientation information to further constrain the model matching process and help discount any spurious edge feature match hypotheses [4, 5]. In the current work, we therefore also account for the predicted orientations of edge pixels in our model matching procedure. In contrast to other work, however, our Likelihood distributions and scale factors are defined by measurement of the appropriate distributions.

## 2.2 Lateral Feature Shifting

Further consideration is required for representing the ‘fixed’ type geometrical features as described above. For many man-made objects, inter-surface regions will be slightly curved in nature. The positions of any corresponding edges will vary within these regions depending on the relative illumination. General behaviour can be predicted from simple illumination models. For distant illumination, proximal parts of an otherwise homogeneous surface at the same orientation are expected to have equivalent grey-level values. This ensures that any spatial effects apply systematically along an extended boundary, i.e. a lateral shift. Observations of these problems have led us

to conclude that most of the errors of re-projection resulting from simple wireframe models correspond to such displacements (see Figure 1).

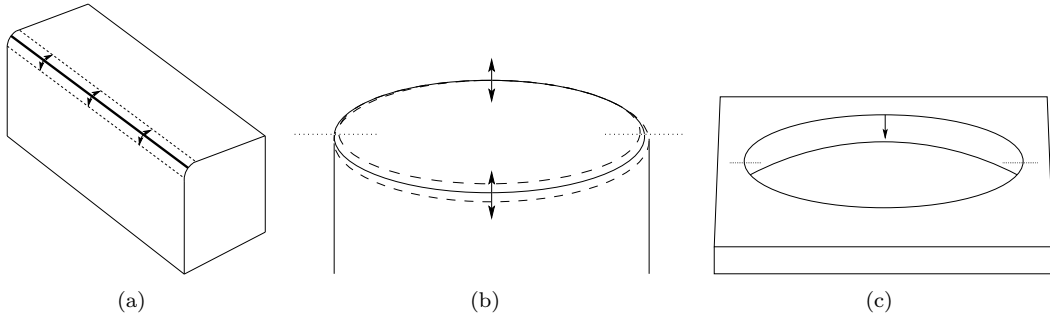


Figure 1: *The various sources of lateral feature shifting due to lighting and modelling simplifications; the location of a straight edge on a curved boundary (a), the end of a cylinder (b) and a hole in a thin plate (c). Circular features, visibly projected as ellipses, can simply be split along their major axis, with each half being allowed to shift independently.*

Our software has been adapted to optimise the lateral locations of features in the image plane, so that the most likely position of each feature can be determined. This modification is essential if we intend to quantitatively interpret any resulting Likelihood scores. This strategy is also used to counter any imprecision of the manual wireframe construction process or the edge feature detection process. Without this mechanism, the location process may be dominated by systematic shifts in the extended features, with alignment driven to balance a variety of artifacts. This illumination dependency does not appear to have been considered previously in the computer vision literature. This is perhaps because any associated effects are unobservable when the object models are not accurate predictions of appearance, or when the objects chosen for demonstration are 2D or sharply defined simple geometrical shapes [3].

### 2.3 Quantitative Feature Localisation

In contrast to previous approaches to the localisation of structure (which seem to us to not be based on recognisable models of image formation), we have attempted to construct a fully quantitative Likelihood scheme, which accounts for the distributions of feature localisation and orientation accuracy observed in data. The approach requires that our Likelihood models are derived from the probability of detecting features with specific measured properties. These models then support the construction of hypothesis tests for feature detection.

Assuming that the spatial location and orientation of an edge are uncorrelated (which is true for sufficiently large separations between projected points), and given a wireframe model and a set of camera parameters  $\theta$ , we can define the probability of an edge pixel being present at a certain location  $(x, y)$ , with an orientation  $\psi$  and within intervals  $\Delta_x \Delta_y \Delta_\psi$  (since we are initially dealing with probability densities) as

$$\begin{aligned}
 P(x, y, \psi | \theta) &= \\
 \int_{x-\Delta_x}^{x+\Delta_x} \int_{y-\Delta_y}^{y+\Delta_y} \int_{\psi-\Delta_\psi}^{\psi+\Delta_\psi} p(x, y | \theta) * p(\psi | x, y, \theta) dx dy d\psi & \quad (1) \\
 \approx p(x, y | \theta) * p(\psi | x, y, \theta) 2\Delta_x 2\Delta_y 2\Delta_\psi &
 \end{aligned}$$

Taking the negative logarithm

$$\begin{aligned}
 -\ln P(x, y, \psi | \theta) &= -\ln p(x, y | \theta) - \ln p(\psi | x, y, \theta) & (2) \\
 &= -\log(8\Delta_x \Delta_y \Delta_\psi)
 \end{aligned}$$

In order to unambiguously define this probability, we must have a methodology for selecting appropriate intervals. We choose to define the intervals proportional to the measurement accuracy ( $\text{var}(x)$ , etc.). We will see below how this regenerates conventional statistical measures. So

$$\begin{aligned}
 -\ln P(x, y, \psi | \theta) &= -\ln p(x, y | \theta) - \ln p(\psi | x, y, \theta) & (3) \\
 &= -\frac{1}{2} \log(\text{var}(x)\text{var}(y)\text{var}(\psi)) + \text{constant}
 \end{aligned}$$

We now need to be specific regarding how these probabilities are computed.

The edge location term ( $-\ln p(x, y|\theta)$ ) can be defined as the probability that the local information measure (edge strength) will be larger than some number of its neighbours. On the assumption of approximate uniform Gaussian errors on the edge strength values, the probability that the pixel under consideration would be larger than one of its neighbours would be computed using the ‘erf’ function, which can be approximated using a linear ‘ramp’ function. The probability that the value considered will be bigger than a sample of values from the local neighbourhood can then be approximated using a ‘rank’ filter process [6].

Using error propagation, it is possible to show that the estimated error on the local (arctan-based) edge orientation measurement is inversely proportional to the edge strength (summed squared derivative  $r^2$ ) and proportional to the image noise, i.e.

$$\text{var}(\psi) \approx \frac{\delta^2}{r^2} \quad (4)$$

The orientation term for equation (2) can be selected in order to match the error distribution on orientation measurement ( $\text{var}(\psi)$ ), with  $\phi(x, y)$  representing the edge orientation of a feature pixel

$$\begin{aligned} -\ln p(\psi|x, y, \theta) &= \frac{(\phi(x, y) - \psi(x, y, \theta))^2}{2 \text{var}(\psi)} \\ &+ \frac{1}{2} \log(2\pi \text{var}(\psi)) \end{aligned} \quad (5)$$

When this is substituted into the log probability, the second term in equation (5) effectively cancels with the interval for the angle measurement  $\Delta\psi$  to give a chi-square like statistic<sup>1</sup>. The two remaining variance terms can be combined with other constant factors which play no further role in the estimation of parameters or parameter variances.

Unfortunately, the above measure contains a subtle but important problem. Valid use of Likelihood requires that we do not change the interval relating probability density to probability during the process of parameter estimation. One way to understand this is to observe that if we instead define the interval using an estimate of variance at each location in the image, then we will get perfectly good statistical matches to data in regions of high variance (i.e. featureless noisy surfaces). Simply optimising this function is likely to locate a curve over featureless regions<sup>2</sup>. Specifically, in this case, we need to know the true values of  $\text{var}(\psi)$  for the optimal location of the curve before we have found that location. A priori knowledge of the expected orientation error at the solution would allow us to make a comparison for the degree of match between model and scene orientation that is informed by our knowledge of the expected level of conformity for good data. We can avoid the problem of lack of knowledge of  $\text{var}(\psi)$  at the solution by observing that, for a wide range of edge strengths, the expected orientation error is approximately constant and scales with the intrinsic image noise, so that  $\text{var}(\psi) \approx \kappa^2\delta^2$  where  $\delta$  is the estimated pixel value error.

The combined Likelihood  $L$  (which is related to the previously defined probability by an arbitrary constant) can be written as

$$\begin{aligned} \ln L(x, y, \psi, \theta) &= V(x, y) + \frac{(\phi(x, y) - \psi(x, y, \theta))^2}{2 \kappa^2\delta^2} \\ &= -\ln P(x, y, \psi|\theta) + \text{constant} \end{aligned} \quad (6)$$

Slight modification of this approach is necessary to deal with occlusion and specularly. Here, we use residual truncation (setting a maximum value  $\chi_{max}^2$  for the contribution to the cost function from each data point).

As the Likelihood is related directly to the quantitative probability of observing the data by a constant, we believe that this satisfies the requirements necessary for the optimisation to result in a ‘consistent’ estimate of the required parameters. The resultant cost for each of a feature’s pixels can then be summed to give a combined score for use in alignment.

The key observation regarding the above approach is that we have well defined predictions for the expected behaviour of the orientation and location distributions. The approximate behaviour of the orientation term is that of a Gaussian distribution with width  $\kappa\delta$ . As the process of edge location is a form of integral transform (i.e. analogous to histogram equalisation), the approach should deliver a uniform distribution of probabilities for true

<sup>1</sup>If we do not choose the interval scaling in the way we have suggested here, and for example choose to ignore it and apply the conventional definition of Likelihood as provided in standard texts [7], then we need to explain where the probability density normalisation terms disappeared to in the construction of standard statistical measures (i.e. squared difference divided by variance).

<sup>2</sup>This is not because these measures are fundamentally wrong, it is just that the statistical tests computed at each location are not suitable for relative comparison.

edges. The log probability for localisation (represented as a potential image  $V(x, y)$ ) should therefore have an exponential distribution.

Given such a quantitative metric for evaluating the predicted locations of wireframe edge features, we are able to conduct further statistical tests to aid our interpretations of the model matching process. Statistical hypothesis tests are implemented in associated research to verify any model match hypotheses and to support determination of feature visibility. This has been illustrated in a previous publication [6]. This also conveniently supports assessment of individual feature visibility. In particular, the Fisher information associated with spatial localisation can be determined, thus supporting focus of computational resources around these ‘key’ features. This information can be stored in the visibility data files, along with constraints on allowable degrees of lateral feature shifting, thus supporting a framework for the automated learning of expected shape variability.

### 3 Results

Objects for this study were selected in order to cover a range of modern fabrication processes and materials. A view dependent wireframe model was constructed for each and the object was initially located in the left image of a stereo pair using a 3D stereo vision system. The object location was then refined using the Likelihood optimisation procedure defined above, with and without lateral shifting of curves and lines. Quantitative hypothesis tests were performed for each wireframe curve as described in [6]. Each object curve was classified as present if more than 45% of its projected points had a hypothesis probability greater than 0.01%.

The utility of the process of lateral feature shifting can be demonstrated by assessing the effects on model match feature verification. Model matching was performed on each of the 8 test objects for 3 different views with and without lateral feature shifting. Given a valid model match, in each case, the number of features composing the curves verified as being present from those predicted were recorded. The cumulative results therefore indicate the proportion of edge features which require lateral shifting in order to be brought into quantitative alignment with corresponding image evidence. These results are detailed in Table 1.

In the general case, it can be observed from Table 1 that lateral feature shifting significantly affects the outlined verification strategy, i.e. the predicted positions of the edge features. There is nearly a 9% average gain in the proportion of predicted features verified as being present across the objects, though some objects and feature types can be seen to be more affected. There is no observed difference for one of the first views of the first object, since for this (rear) projection the features are very sparse and sharply defined. The object displayed in Fig. 2(i) appears perhaps the most affected - although this might not appear the best candidate for such processes, the object was found to be physically distorted from the rigid wireframe model and the lateral feature shifting helped to alleviate some of the associated modelling inadequacies. The relatively small proportion of identified features for two of the views of object (g) can be attributed to the features modelled across the back of the object, for which there was insufficient contrast for detection.

Observation of the instances where lateral feature shifting affects the imaged locations of features supports the initial hypothesis that fixed-feature wireframe models are not sufficient to support accurate quantitative alignment and verification. The process is required to accommodate shifts in the relative positions of extended (blunt) edge features due to orientation-illumination dependencies. Additionally, the process supports the modelling of thin planar surfaces with a single boundary. In general, lateral feature shifting is a robust strategy which compensates for many inaccuracies introduced by the modelling process.

### 4 Conclusions

The aim of this paper has been to describe a strategy for constructing 3D wireframe models to represent a broad range of rigid, man-made 3D objects in support of object detection and localisation tasks. A technique to incorporate viewpoint dependent occluding boundaries for structures such as conical sections has been implemented. Additionally, viewpoint dependency files have been proposed as a convenient and direct way to account for feature visibility across the view sphere. We believe this approach to be more suited to a vision system that must learn incrementally, potentially from small numbers of views, than a CAD approach which requires the accurate and complete reconstruction of surfaces. Wireframe modelling has previously been restricted to objects composed of relatively fixed edge features. In allowing lateral feature shifting, our approach can be understood as an appearance model for detectable edge structure. In conjunction with the outlined modelling strategy, the main issue addressed by this work is the dependency upon accurate quantitative alignment of such edge-based wireframe models.

In allowing lateral shifting, we have arrived at a process which, contrary to the dominant approach to object location and validation in the literature, does not base statistical decisions regarding the presence or absence of

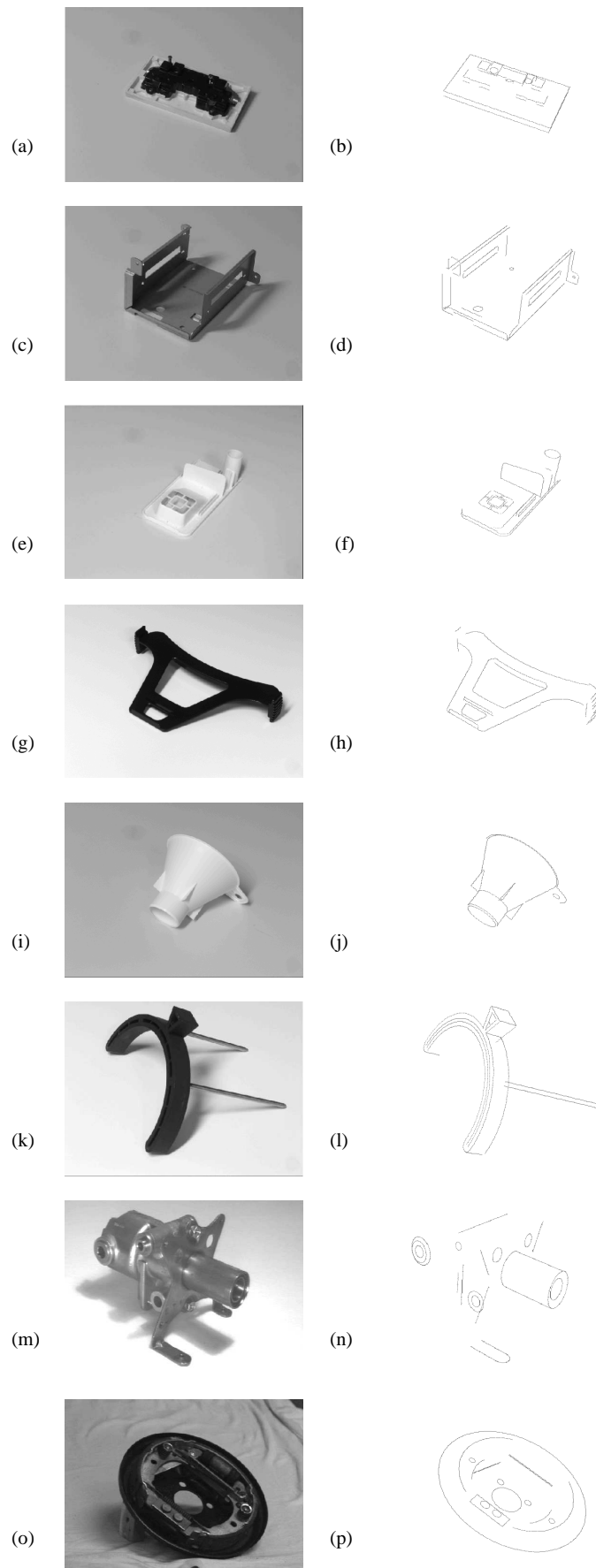


Figure 2: *Test objects and the features re-projected from the corresponding view-based 3D models.*

Object Model (as Fig. 2)	Total Model Points	Points Verified with Lateral Feature Shift	Points Verified without Lateral Feature Shift	% Diff.
(a) (b) View 1	943	743 (78.8%)	704 (74.7%)	4.1
(a) (b) View 2	837	817 (97.6%)	817 (97.6%)	0.0
(a) (b) View 3	848	832 (98.1%)	794 (93.6%)	4.5
(c) (d) View 1	1247	1172 (94.0%)	1101 (88.3%)	5.7
(c) (d) View 2	1384	1346 (97.3%)	1286 (92.9%)	4.4
(c) (d) View 3	1246	1205 (96.7%)	1176 (94.4%)	2.3
(e) (f) View 1	816	740 (90.7%)	620 (76.0%)	14.7
(e) (f) View 2	727	596 (82.0%)	527 (72.5%)	9.5
(e) (f) View 3	767	639 (83.3%)	592 (77.2%)	6.1
(g) (h) View 1	1069	940 (87.9%)	888 (83.1%)	4.8
(g) (h) View 2	1361	829 (60.9%)	759 (55.8%)	5.1
(g) (h) View 3	1306	849 (65.0%)	689 (52.8%)	12.2
(i) (j) View 1	657	493 (75.0%)	385 (58.6%)	16.4
(i) (j) View 2	634	485 (76.5%)	388 (61.2%)	15.3
(i) (j) View 3	683	639 (93.6%)	559 (81.8%)	11.8
(k) (l) View 1	1142	1076 (94.2%)	888 (77.8%)	16.4
(k) (l) View 2	1342	1302 (97.0%)	1165 (96.8%)	10.2
(k) (l) View 3	1378	1220 (88.5%)	1170 (84.9%)	3.6
(m) (n) View 1	1230	118 (90.9%)	979 (79.6%)	11.3
(m) (n) View 2	1224	1016 (83.0%)	903 (73.8%)	6.2
(m) (n) View 3	1156	1012 (87.5%)	836 (72.3%)	15.2
(o) (p) View 1	1418	1393 (98.2%)	1285 (90.6%)	7.6
(o) (p) View 2	1547	1547 (100.0%)	1365 (88.2%)	11.8
(o) (p) View 3	1563	1533 (98.1%)	1137 (72.7%)	15.4

Table 1: *The proportion of predicted edge feature points verified as being present with and without a process of lateral feature shifting.*

features upon detailed predictions of location. Instead, we use only approximate information regarding location, and statistical tests are based upon a quantitative model of feature detection. This model follows directly from the Likelihood theory used to define the presence and orientation of features (i.e. we do not need arbitrary weighting factors to accommodate location and orientation terms in our cost function).

Unconstrained lateral shifting raises a possible problem for objects composed entirely of lines, as it prevents a unique definition of object location (e.g. a centroid). This issue can be resolved either with use of 2nd order curves, or through the use of a localisation constraint during positional optimisation. Such a constraint can be obtained from accumulated statistics regarding the apparent position of features detected in a series of equivalent views and simply combined into the existing framework. In essence we could learn which features are ‘fixed’ at their expected locations. Based upon our results we would predict that many features ( $\sim 70\%$ ) are already well projected without the need for significant shifting. However, the modifications and experiments needed to support this were beyond the scope of the current work.

Our approach involves trying to model the features that can be practically extracted from images and their allowed variability in scenes. We propose that this is well suited to the task of extracting quantitative information from images in the context of a general system that must learn. Though this avoids surface modelling, we expect that it will be possible to infer some qualitative surface information from multiple views of an object under variable illumination later, once basic competences for recognition and location of objects and features have been established. We therefore move the requirement to model surfaces out of the representation for view-based recognition and location and leave this aspect of scene interpretation for later. We suggest that surface identification in an image should proceed by recognition of scene contents followed by testing of image data with regard to generated surface hypotheses. This would lead to surface detection on the basis of an *absence* of quantitative evidence which invalidates the hypothesis, rather than bottom up measurement based upon the *presence* of detectable features.



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